Potenzen 1

1a) $\lg(a+b) = 2$

$$\lg(a-b) + \lg\sqrt{a+b} - \lg\frac{a^2-b^2}{a^2+2ab+b^2}$$

$$\lg\left((a+b)^{1/2}\right) \qquad \lg\frac{\cancel{(a+b)}(a-b)}{\cancel{(a+b)}(a+b)}$$

$$\tfrac{1}{2}\lg(a+b) \qquad -\left(\lg(a-b) - \lg(a+b)\right)$$

$$\Rightarrow \lg(a-b) + \tfrac{1}{2}\lg(a+b) - \lg(a-b) + \lg(a+b) = 1{,}5\cdot\lg(a+b) = \underline{\underline{3}}$$

1b) $f(x)$, $g(x)$, beide: $y = a\cdot x^r$

$P(2/8)$ auf f
$Q(1/\tfrac{1}{2})$ " f
$R(-\tfrac{1}{2}/8)$ auf g
$S(-1/2)$ " g

$$f(x): \left|\begin{array}{l} 8 = a\cdot 2^r \\ \tfrac{1}{2} = a\cdot 1^r \end{array}\right| \Rightarrow a = \tfrac{1}{2}$$

$$\Rightarrow \underline{\underline{f(x) = \tfrac{1}{2}x^4}}$$

$$\Rightarrow \frac{8}{1/2} = 2^r \Rightarrow 16 = 2^r \Rightarrow 2^4 = 2^r \Rightarrow r = 4$$

$$g(x) \left|\begin{array}{l} 8 = a\cdot(-\tfrac{1}{2})^r \\ 2 = a\cdot(-1)^r \end{array}\right| \Rightarrow a = \frac{2}{(-1)^r} = 2(-1)^{-r} = 2\left(2\cdot(-\tfrac{1}{2})\right)^{-r} = 2(2)^{-r}(-\tfrac{1}{2})^{-r}$$

$$\Rightarrow 8 = 2(-1)^{-r}\cdot(-\tfrac{1}{2})^r$$

$$8 = 2(2)^{-r}\cancel{(-\tfrac{1}{2})^{-r}}\cdot\cancel{(-\tfrac{1}{2})^r} \Rightarrow 8 = 2(2)^{-r} \quad |\cdot 2^r, :8$$

$$\Rightarrow 2^r = \frac{2}{8} = \frac{1}{2^2} = 2^{-2} \Rightarrow r = -2 \Rightarrow \underline{\underline{g(x) = 2x^{-2}}}$$

$$\Rightarrow a = 2(-1)^{-(-2)} = 2(-1)^2 = 2$$

$f(x) = g(x)$ → muss laut Grafik 2 Lösungen haben:

$$\frac{x^4}{2} = \frac{2}{x^2} \quad |\cdot x^2, \cdot 2 \Rightarrow x^6 = 4 \Rightarrow x_{1/2} = \pm\sqrt[6]{4} = \pm(2)^{\frac{2}{6}} = \pm(2)^{\frac{1}{3}} = \pm\sqrt[3]{2}$$

$$\Rightarrow S_{1/2}: \frac{(2^{1/3})^4}{2^1} = 2^{4/3}\cdot 2^{-3/3} = 2^{1/3}$$

$$\frac{(-2^{1/3})^4}{2^1} =$$

$$\Rightarrow \underline{\underline{S_1\left(\sqrt[3]{2} / \sqrt[3]{2}\right),\ S_2\left(-\sqrt[3]{2} / \sqrt[3]{2}\right)}}$$

Potenz 2, 3

2 a) $\frac{(21^2 \cdot a^3)^5}{(7^5 \cdot a^4)^2}$; $a \neq 0$ $\Rightarrow$ $\frac{21^{10} a^{15}}{7^{10} a^8} = \left(\frac{21}{7}\right)^{10} a^7 = 3^{10} a^7$

b) $\lg(a^2-1) - \lg(a-1) - \lg((a+1)^2)$

$\lg[(a+1)(a-1)] - \lg(a-1) - 2\lg(a+1) = \lg(a+1) + \lg(a-1) - \lg(a-1) - 2\lg(a+1) = -\lg(a+1)$

$= \lg\left(\frac{1}{a+1}\right)$

c) $2^x 3^{x+1} = \frac{1}{2} = 2^{-1}$ $\Rightarrow$ $2^x \cdot 3^x \cdot 3 = 2^{-1}$

$6^x = 6^{-1}$ $\Rightarrow$ $x = -1$

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3 a) $\frac{a^x - a^{x+2}}{a^{x+1} - a^x}$ $\Rightarrow$ $\frac{a^x(1-a^2)}{a^x(a-1)}$ $\Rightarrow$ $\frac{(1+a)(1-a)}{(a-1)} = \frac{(1+a)(1-a)}{(-1)(1-a)} = \frac{1+a}{-1} = -a-1$

b) $\underbrace{2\log_3(2\sqrt{a})}$ $- \log_3(7a) + \log_3\left(\frac{7}{4}\right)$

$\log_3(4a) - \log_3(7a) + \log_3\left(\frac{7}{4}\right)$ $\Rightarrow$ $\log_3\left[\frac{4a \cdot \frac{7}{4}}{7a}\right] = \log_3(1) \Rightarrow 3^x = 1 \Rightarrow x = 0$

c) $x > 0$ $\quad f(x) = a \cdot x^n$ $\quad A(1/4)$, $B(2/\frac{1}{2})$

$\Rightarrow \left| \begin{matrix} 4 = a \cdot 1^n \\ \frac{1}{2} = a \cdot 2^n \end{matrix} \right| \rightarrow a = 4$ $\Rightarrow \frac{1}{8} = 2^n \Rightarrow 2^{-3} = 2^n \Rightarrow n = -3$

$\Rightarrow$ $f(x) = 4x^{-3}$ $\quad$ ∩ mit $x = \frac{1}{2}$ und ∩ mit $y = \frac{1}{16}$

Achtung Fallen! $f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^{-3} = 4 \cdot \frac{1^{-3}}{2^{-3}} = 4 \cdot 2^3 = 4 \cdot 8 = 32$ $\Rightarrow$ $S_1\left(\frac{1}{2}/32\right)$

und $\quad 4x^{-3} = \frac{1}{16}$ $\quad | \cdot x^3, \cdot 16$

$\Rightarrow$ $64 = x^3$ $\rightarrow$ $4^3 = x^3$ $\Rightarrow x = 4$ $\Rightarrow$ $S_2\left(4/\frac{1}{16}\right)$

Potenzen 4, 5

4 a) $\dfrac{a^{-5}b^2}{c^{-2}a^3} : \dfrac{c^4b^3}{ba^8}$ $a, b, c \neq 0$

$$\Longrightarrow \frac{a^{-5}b^2}{c^{-2}a^3} \cdot \frac{ba^8}{c^4b^3} = a^{-5}b^2ba^8c^2a^{-3}c^{-4}b^{-3} = a^{(-5+8-3)}b^{(2+1-3)}c^{(2-4)}$$

$$= a^0 \cdot b^0 \cdot c^{-2} = \underline{\underline{\frac{1}{c^2}}}$$

4 b) $5^{2x} - 4 \cdot 5^x = 0 \Rightarrow 5^{2x} = 4 \cdot 5^x$

$$\Rightarrow (5^x)^2 = 4 \cdot 5^x \quad | :5^x$$

$$\frac{(5^x)^{\cancel{2}}}{\cancel{5^x}} = 4 \Rightarrow \underline{\underline{5^x = 4}} \Rightarrow x = \frac{\log(4)}{\log(5)} = \underline{\underline{0{,}861}}$$

4 c) Menge Wirkstoff 0,05g ≙ 400 Sprühstösse $\longrightarrow \dfrac{0{,}05\,g}{400\,St.} = \underline{\underline{0{,}000125\ g/Stoß}}$

$$= \underline{\underline{1{,}25 \cdot 10^{-4}\ g/Stoß}}$$

Jeder Stoß: $3{,}4 \cdot 10^8$ Partikel Masse: $\dfrac{1{,}25 \cdot 10^{-4}\,g}{3{,}4 \cdot 10^8\,P} = \underline{\underline{3{,}68 \cdot 10^{-13}\ g/Partikel}}$

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5 a) $\dfrac{(a^2c)^2}{a^2c^2+bc^2} - \dfrac{b^2}{a^2+b} \Longrightarrow \dfrac{(a^2+b)(a^4c^2) - (a^2c^2+bc^2)(b^2)}{(a^2c^2+bc^2)(a^2+b)}$

$$= \frac{a^6c^2 + a^4bc^2 - a^2b^2c^2 - b^3c^2}{a^4c^2 + a^2bc^2 + a^2bc^2 + b^2c^2} = \frac{\cancel{c^2}(a^6 + a^4b - a^2b^2 - b^3)}{\cancel{c^2}(a^4 + a^2b + a^2b + b^2)} = \frac{a^6 + a^4b - a^2b^2 - b^3}{(a^2+b)^2}$$

$a^6 + a^4b - a^2b^2 - b^3 : (a^2+b) = a^4 - b^2 = (a^2+b)(a^2-b) \longrightarrow \dfrac{\cancel{(a^2+b)}(a^2-b)}{\cancel{(a^2+b)}}$

$-\ a^6 \quad a^4b$

$-\quad - \quad -a^2b^2 - b^3$

$-\quad -a^2b^2 - b^3$

$\quad - \quad -$

$$= \underline{\underline{a^2 - b}}$$

5 b) $\log_3(x+5) - \log_3(5x+25) + \underbrace{2\log_3(\sqrt{5x})}_{\log_3[(\sqrt{5x})^2] = \log_3(5x)} \Rightarrow \log_3\left[\dfrac{(x+5)\cdot 5x}{(5x+25)}\right] = \log_3\left(\dfrac{x\cancel{(5x+25)}}{\cancel{(5x+25)}}\right)$

$$= \underline{\underline{\log_3(x)}}$$

5 c) $f(x) = cx^n$; $n \in \boxed{?}$ vermutlich

alle $f(x)$ / alle y sind positiv (über x-Achse) $\Longrightarrow c > 0$

$f(0)$ ist nicht definiert, bei hohen $|x|$ wird $f(x)$ immer kleiner $\longrightarrow n < 0$

n ist gerade, da jede gerade Wurzel 2 Lösungen hat: $\pm f(x)$, x ist nur für $x > 0$ zulässig.

Potenzen 6

6a) $$\frac{16a^3 - a}{a^2 - 4a^3} \Rightarrow \frac{\cancel{a}(16a^2-1)}{a^{\cancel{2}}(1-4a)} = \frac{(4a+1)\cancel{(4a-1)}}{-a\cancel{(4a-1)}} = -\frac{4a+1}{a}$$

6b) $2^x + 4^x = 2$ probieren: $2^x + (2\cdot 2)^x = 2^1 \Rightarrow 2^x + 2^x\cdot 2^x = 2^1 \Rightarrow 2^x(1+2^x) = 2^1$

$\Rightarrow 1-2^x = \frac{2}{2^x} \Rightarrow 1-2^x = 2^{1-x} \Rightarrow$ funktioniert nicht ...

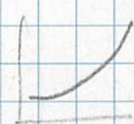
also: austesten $x=-2$, $x=-1$, $x=0$, $x=1$, $x=2$

$2^{-2}+4^{-2}=0{,}3125$ $2^{-1}+4^{-1}=0{,}75$ $2^0+4^0=1$ ✓ $2^1+4^1=6$ $2^2+4^2=20$

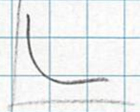
↳ $x=0$

6c) $f(x) = 5\left(\frac{1}{3}\right)^x$ $g(x) = \frac{1}{3}\cdot 2^x$

wächst immer stärker an



wird immer kleiner



P: $5\left(\frac{1}{3}\right)^x = \frac{1}{3}2^x \;\Big|\cdot 3, \cdot 3^x \Rightarrow 15 = 2^x\cdot 3^x \Rightarrow 15 = 6^x \mid \log()$

$\Rightarrow \frac{\log(15)}{\log(6)} = x = 1{,}511$

$\Rightarrow y = \frac{2^x}{3} = \frac{5}{3^x} = 0{,}950$

P(1,511 / 0,95)